

Are Gamma-ray Bursts Universal?

David Eichler¹ & Amir Levinson²

ABSTRACT

It is noted that the Liang-Zhang correlation can be accounted for with the viewing angle interpretation proposed earlier. The Ghirlanda correlation, recently generalized by Nava et al (2006) to a wind profile, can be accounted for by the viewing angle interpretation accordingly generalized to a wind profile. Most of the scatter in the spectra and time-integrated brightness in γ -ray bursts (GRB) can thus be accounted for by variation in two parameters, 1) the viewing angle and 2) the jet opening angle, with very little variation in any other intrinsic parameters. The scatter in apparent isotropic equivalent fluence and other parameters is reduced by a factor of order 30 when each of these parameters is considered. Possible difficulties with alternative explanations are briefly discussed. It is also noted that the relative scatter in the Amati and Ghirlanda correlations suggests certain conclusions about the inner engine.

Subject headings: black hole physics — gamma-rays: bursts and theory

1. Introduction

Several years ago Frail et al. (2001) argued that the γ -ray energy $E_{\gamma,j}$ in γ -ray bursts (GRB) had much less variation than the isotropic equivalent energy $E_{\gamma,iso}$. The hypothesis they implied was that the opening angle of the jet, θ , as determined from the break in the afterglow light curve, was the major factor in determining the isotropic equivalent flux. Dimmer GRB, it was concluded, are dimmer because the same energy is spread out over a larger solid angle. Thus, the quantity $\theta^2 E_{\gamma,iso}/2$ is the true γ -ray energy and this quantity seems to have much less scatter than $E_{\gamma,iso}$, uncorrected for opening angle.

It was also noticed by Amati et al that the isotropic equivalent luminosity $E_{\gamma,iso}$ is to be strongly correlated with the spectral energy peak $h\nu_{peak}$ as $E_{\gamma,iso} \propto (h\nu_{peak})^2$. It was pointed out (Eichler & Levinson 2004) that the Amati relation is what one would expect if

¹Physics Department, Ben-Gurion University, Beer-Sheva 84105, Israel; eichler@bgumail.bgu.ac.il

²School of Physics and Astronomy, Tel-Aviv University, Tel Aviv 69978, Israel; Levinson@wise.tau.ac.il

the reduction in ν_{peak} was an illusion created by the viewer not being in the direction of the jet itself. If the viewing offset, $\Delta = \theta_{obs} - \theta$, is only a small fraction of the jet opening angle, viz., $\Delta \ll \theta$, then the viewer sees contributions from a solid angle of order Δ^2 and a spectral energy peak of $h\nu_{peak} = h\nu^* \mathcal{D}$, where $\mathcal{D} = (1 - \beta \cos \Delta)^{-1} \simeq 2/\Gamma \Delta^2$ is the Doppler factor of the fluid element closest to the observer. The Amati correlation then follows because $E_{\gamma,iso}$ is proportional to D^2 (see appendix A in Levinson & Eichler, 2005). This explanation of the Amati et al relation would not apply to a pencil beam (Lamb et al., 2004) or a solid filled in beam (Yamazaki et al. 2004), for it makes the key assumption that the solid angle of jet that contributes to the observed GRB luminosity is proportional to the square of the viewing angle offset. It is consistent with the observed relative frequencies of GRB and X-ray flashes only if the jet has a nontrivial geometry so that a large fraction of all viewers are rather close to the perimeter of the jet.

Subsequently, Ghirlanda et al. (2004) reported that $E_{\gamma,j}$ correlates with ν_{peak} as $E_{\gamma,j} \propto \nu_{peak}^{1.5}$. The implication was that the solid angle, namely the ratio of $E_{\gamma,j}$ to $E_{\gamma,iso}$, is itself correlated with these two quantities. Levinson & Eichler (2005) then noted that the modest difference between the Amati et al relation and the Ghirlanda relation could be accounted for in a natural way without making any assumption of correlation between the physical opening angle and the jet energy output. (Because this difference is modest and comparable to the scatter, we regard our explanation of it as reasonable but preliminary, pending a larger data set of GRB with known redshifts, peak energies, and break times.) The difference between the respective exponents in each relation is naturally accounted for by the fact that the estimate of the jet opening angle is itself weakly affected by viewing angle. Although the inferred jet opening angle $\theta_{j,inf}$ is only very weakly dependent on E_{iso} , as $\theta_{j,inf} \propto E_{iso}^{-1/8}$, an offset viewer would nonetheless overestimate $\theta_{j,inf}$ because his offset viewing angle causes him to underestimate the true fluence $E_{\gamma,iso}$ of the jet. The overestimate of the solid angle $\pi\theta_{j,inf}^2$, is therefore proportional to $E_{\gamma,iso}^{-1/4}$ i.e. to $\nu_{peak}^{-1/2}$, and this is precisely the difference in the exponents in the Amati and the Ghirlanda correlations. (The quantity E_{iso} as it appears in the expression for the inferred jet opening angle is, in fact, the kinetic energy of the jet per unit solid angle. It was assumed to be proportional to $E_{\gamma,iso}$ in the above analyses to within a constant.) The two observed relations thus provide confirmation that the conclusions of Frail et al., Amati et al. and Ghirlanda et al. not only consistent but mutually supporting. That is to say, the residual scatter in $E_{\gamma,j}$, after making the correction for opening angle variation, is mostly accounted for by making a viewing offset angle correction and vice versa. These two corrections together eliminate most of the variation in both E_{iso} and $E_{\gamma,j}$ and they even reduce the scatter in the inferred opening angle $\theta_{j,inf}$. The fact that applying only one of these two corrections leaves residual scatter is by no means evidence against its validity. It merely implies that two separate factors influence the measured γ -ray fluence $E_{\gamma,iso}$.

Eichler and Jontof-Hutter (2005) noted that the γ -efficiency ϵ_γ , defined to be the ratio of γ -ray energy to baryon kinetic energy (estimated from the X-ray afterglow luminosity at an observer time t of 10 hours) to correlate with ν_{peak} as $\epsilon_\gamma \propto \nu_{peak}^{3/2}$. This is nearly the same correlation exponent as in the Ghirlanda relation. The implication is that as ν_{peak} is decreased, the observed γ -ray fluence decreases with ν_{peak} much faster than the blast energy. This is easily understood in the viewing angle interpretation of the Amati/Ghirlanda correlations, because the true blast energy does not depend on the viewing angle. Moreover, an off-set observer should see suppressed afterglow until the blast has decelerated enough to encompass the observer in the $1/\Gamma$ emission cone of the blast material that generates the afterglow. This effect is consistent with observations that have been interpreted (Eichler 2005; Eichler and Granot 2006) as delayed afterglow onset such as gaps between the prompt emission and the apparent beginning of the afterglow emission (Piro et al. 2005; Nousek et al. 2005 and references therein).

Explanations of the Amati and Ghirlanda correlations that posit a true physical dependence of GRB energy on spectral peak where both vary considerably (e.g. Rees and Meszaros 2005) leave unanswered the question of why the GRB energy should have a range of several orders of magnitude while the blast energy over the same data set shows a far less noticeable variation. If the γ -ray photosphere is controlled by (the electron counterpart of) a baryonic component, then the blast energy might be expected to vary at least as much as the γ -ray energy, because radiative energy is transferred to the kinetic energy of the baryons during the adiabatic expansion below the photosphere. It may, of course, be that the photosphere is controlled by pairs (e.g. Eichler 1994; Eichler and Levinson 2000), a possibility seriously considered by many other authors as well, and that the difference between bright γ -ray bursts and dim ones is expressed primarily by the emission from a pair dominated photosphere, but the question of how the baryon dependence scales with burst energy would be left open.¹

Liang and Zhang (2005) found the following relation:

$$E_{\gamma,iso,52} = (0.85 \pm 0.21) \left(\frac{h\nu_{peak}}{100 \text{ KeV}} \right)^{1.94 \pm 0.17} t_{break,d}^{-1.24 \pm 0.23}, \quad (1)$$

where $E_{\gamma,iso,52}$ is the observed isotropic equivalent γ -ray luminosity in units of 10^{52} erg/s, $h\nu_{peak}$ is the peak energy as usual, and $t_{break,d}$ is the break time of the afterglow light curve

¹The objection is sometimes raised that a pair dominated photosphere could not have a non-thermal photon spectrum, however, we see no reason why not. See, for example, Blandford and Payne 1982; Eichler 1994. Moreover, non-thermal radiation can be generated in the optically thin region independently of radiation from the photosphere.

measured in days. All relevant quantities are measured in the cosmological rest frame. A similar relation was found later by Nava et al. (2006) using a different method. As pointed out by Nava et al. (2006), both results appear to be consistent, within the errors, with

$$E_{\gamma,iso} \propto (h\nu_{peak})^2 t_{break}^{-1}. \quad (2)$$

This expresses a scatter in the Amati relation that follows from the scatter in the inverse break time. Note that $E_{\gamma,iso}$ is less for an offset observer than the "true" E_{iso} for an observer in the beam. Below, we offer a simple explanation for this relation.

Consider a conical jet of kinetic energy E_j , Lorentz factor Γ and semi-opening angle θ , expanding into an external medium of density $n(r) = \kappa r^{-d}$, where κ is some constant and r is the distance from the center of the explosion. In the adiabatic regime the total energy is conserved, and the evolution of the gas behind the forward shock is given by (e.g., Meszaros et al. 1998)

$$E_j \propto \kappa \Gamma^2 r^{3-d} \theta^2. \quad (3)$$

In terms of the observer time, $dt = dr/\Gamma^2$, and the jet isotropic equivalent energy, defined as $E_{iso} = \theta^{-2} E_j$, we have

$$\Gamma(t) \propto \left(\frac{E_{iso}}{\kappa} \right)^{1/(8-2d)} t^{(d-3)/(8-2d)}. \quad (4)$$

Let t_j denote the time at which $\theta = \Gamma^{-1}$. Using the last equation we obtain

$$\theta \propto \left(\frac{E_{iso}}{\kappa} \right)^{-1/(8-2d)} t_j^{(3-d)/(8-2d)}, \quad (5)$$

and

$$E_j = \theta^2 E_{iso} / 2 \propto \kappa \left(\frac{E_{iso} t_j}{\kappa} \right)^{(3-d)/(4-d)}. \quad (6)$$

Now, assume that a fraction η_γ of the kinetic energy is emitted as gamma rays. (We allow for the possibility that η_γ is greater than 1 and in this regard E_j should be distinguished from the total energy which is the sum of the kinetic and radiative energy.) The observed isotropic γ -ray energy measured by an observer observing the source at some viewing angle outside the jet that corresponds to an observed peak energy $h\nu_{peak}$ is $E_{\gamma,iso} \propto \eta_\gamma E_{iso} (\nu_{peak}/\nu^*)^2$, where $h\nu^*$ defines the spectral peak energy that will be measured by an on-axis observer (Eichler & Levinson 2004; Levinson & Eichler 2005). By employing eq. (6) to eliminate E_{iso} , we finally arrive at:

$$E_{\gamma,iso} \propto \eta_\gamma E_{iso} (\nu_{peak}/\nu^*)^2 \propto \eta_\gamma \kappa^{1/(d-3)} E_j^{(4-d)/(3-d)} t_j^{-1} (\nu_{peak}/\nu^*)^2. \quad (7)$$

By associating the observed break time of the afterglow emission with t_j as commonly done, viz., $t_{break} = t_j$, and assuming t_{break} to be independent of viewing angle (as is the case when the observer is within the $1/\Gamma$ emission cone of the afterglow by the time of the break), we conclude that relation (7) is consistent with the Liang/Zhang relation, as given in eq. (2), provided the quantity $\eta_\gamma \kappa^{1/(d-3)} E_j^{(4-d)/(3-d)}$ is universal. For $d = 0$, this is close to stipulating that $\eta_\gamma E_j = E_{\gamma,j}$ is universal.

Next, consider the collimation corrected energy. The jet opening angle $\theta_{j,inf}$ inferred by an off-axis observer that measures isotropic γ -ray energy $E_{\gamma,iso}$ and break time t_{break} , and who assumes ambient medium with density profile as above, satisfies

$$\theta_{j,inf} \propto \left(\frac{E_{\gamma,iso}}{\kappa} \right)^{-1/(8-2d)} t_{break}^{(3-d)/(8-2d)}. \quad (8)$$

The collimation corrected energy that will be obtained by using the latter expression for the jet semi-opening angle is then

$$E_{\gamma,inf} = \theta_{j,inf}^2 E_{\gamma,iso} \propto \eta_\gamma^{(5-d)/4-d} E_j (\nu_{peak}/\nu^*)^{(6-2d)/(4-d)}, \quad (9)$$

which, for a universal $\eta_\gamma^{(5-d)/4-d} E_j$, is consistent with the Ghirlanda relations for both a uniform density medium ($d=0$) and a wind profile ($d=2$), as discussed in Nava et al. (2006). This is not surprising, since the connection between $E_{\gamma,iso}$ and $E_{\gamma,inf}$ is defined in Nava et al (2006) in the same way as here. Regardless of what one assumes about the surrounding density profile, the point remains (as already noted by Nava et al with different phraseology) that the Amati correlation and the Frail correlation between $E_{\gamma,iso}$ and $\theta_{j,inf}^{-2}$ imply the Liang and Zhang correlation given the standard assumptions of afterglow theory. Moreover, the equivalence of the Amati correlation and Ghirlanda correlation if the former is attributed to viewing angle effects (Levinson and Eichler, 2005) is independent of assumptions about the surrounding density profile. The small scatter in the Ghirlanda and Liang/Zhang relations indicates that the “true” GRB energy is universal, as claimed originally by Frail et al (2001).

2. Conclusions and Further Discussion

As noted by Nava et al (2006), the Ghirlanda correlation and its physical implications change with assumption about the surrounding density profile. This is because, unlike the Amati correlation, the Ghirlanda correlation is not one of purely observed quantities, but rather includes within it a theoretical inference about the GRB jet opening angle that depends on assumptions regarding the evolution of the blast wave. In particular, they note that if they assume a wind-like profile, (together with the tacit assumption that opening

angle is uncorrelated with $E_{\gamma,j}$) then the GRB energy $E_{\gamma,j}$ scales linearly with $h\nu_{peak}$, and the photon entropy is constant among the different bursts, whereas this conclusion would not follow if a constant ambient density profile were assumed. The question would remain open as to why the photon entropy would remain constant over a wide range of $E_{\gamma,j}$ and $h\nu_{peak}$, especially if the latter is established at a pair-dominated photosphere.

Here we have shown that the viewing angle interpretation of both the Amati and Ghirlanda correlations, and the equivalence between the two is independent of assumptions about the ambient density profile. This is because the universality of the GRB energy E_j implied by this equivalence is a physically separate issue from the opening angle (the latter presumably established by collimation well downstream of the central engine), and is therefore unaffected by it. A set of GRBs with identical E_j could be placed in an environment of any density profile and the theoretical values of E_j , if correctly inferred by making the correct assumptions about the density profile, would all yield the same conclusion - that the range of E_j is narrow.

That the Ghirlanda relation shows less scatter than the Amati correlation is significant in the same way that the Frail correlation (for a limited range of spectral peak) is. We interpret it to mean that modest variation in the opening angle introduces additional scatter into the observed $E_{\gamma,iso}$ after either $E_{\gamma,j}$ or E_j has been established by the central engine. This is to be contrasted with the reverse situation: that E_{iso} in an outflow is established by the central engine and the γ -ray output $E_{\gamma,j}$ is established, say, by internal shocks whose effective covering solid angle or overall efficiency varies from one GRB to the next. In the latter case, one would expect more scatter in $E_{\gamma,j}$ than in E_{iso} due to the additional scatter in the covering angle. The low scatter $E_{\gamma,j}$ is consistent with, and perhaps even supportive of the claim (Eichler and Jontof-Hutter 2005, Eichler and Granot 2006) that the γ -ray efficiency is close to 100 percent in GRBs, and that only a small fraction of the energy is in blast energy, since this is a reliable way of limiting the γ -ray efficiency to a narrow range.

If the Ghirlanda correlation indeed proves to have a different slope from the Amati correlation this will also be significant. It would imply that the inferred opening angle varies systematically with $h\nu_{peak}$. At present, the implied systematic variation is only comparable to the scatter in solid angle inferred from the observed break times. If there were a wide range of physical jet energies and true spectral peaks, the difference in slope would then be a considerable spread in opening angles associated with the wide range of $h\nu_{peak}$. Specifically, if the range of $h\nu_{peak}$ is from 30 KeV to 1 MeV, and, as assumed by Nava et al (2006), the ambient density is wind-like, then the Ghirlanda correlation would be $E_j \propto h\nu_{peak}$ and it would then follow that $\theta_{inf}^{-2} \propto h\nu_{peak}$. It would follow that the range of solid angles is about 30, scaling in inverse proportion to $h\nu_{peak}$.

We also note that even though E_{iso} does not appear to have much remaining scatter after the various correlations discussed here are accounted for, L_{iso} , the isotropic equivalent luminosity does, because the durations of long bursts vary from several seconds to several hundreds of seconds. Any physical mechanism that ties E_{iso} to $h\nu_{peak}$ would have to tolerate the large variation in GRB duration and the attendant variation in L_{iso} for a given E_{iso} . This is significant because the bulk Lorentz factor at the photosphere, which is likely to enter into $h\nu_{peak}$ in some models, is more likely to depend on L_{iso} than on E_{iso} .

Yet another significant statistic, in our view, is that the blast energy does *not* correlate nearly as noticeably with $h\nu_{peak}$ as does $E_{\gamma,inf}$. In fact, the best fit for the ratio $E_{\gamma,inf}/E_k$ has it correlating linearly as $h\nu_{peak}^{1.4}$ (Eichler and Jontof-Hutter, 2005) which is nearly exactly the Ghirlanda relation [Here $E_{\gamma,inf}$ is the inferred γ -ray output and E_k is the kinetic energy of the ejected mass as inferred from the 10 hour X-ray afterglow (Freedman and Waxman 2001, Lloyd-Rhonning and Zhang 2004). Note that E_k is often used interchangeably with the quantity E_j as defined in Equation 3.] If the Amati correlation were to be attributed to real physical variations in both E_{iso} and $h\nu_{peak}$ that are closely tied together, then it would suggest that bright, hard GRB are brighter than dim, soft ones not primarily because of more baryon kinetic energy, but rather because of greater dominance of other forms of energy. Presumably, the non-baryonic energy is mostly photons and pairs; the point is that it generates more photons without generating noticeably more afterglow. This would be consistent with the best estimate of $E_{\gamma,inf}/E_k$ for the brightest bursts that is considerably greater than unity (Eichler and Jontof-Hutter 2005, Eichler and Granot 2006). However, in the simplest model of an adiabatically expanding baryon-free fireball that expands from a fixed dissipation radius R_0 , the photon entropy is proportional to $E_{tot}^{3/4}$ where E_{tot} is the total energy. So the variation of R_0 with E_{tot} would have to be tailored to obtain a fit with the Ghirlanda correlation, which, for a wind-like ambient density profile, gives a constant photon entropy.

To conclude, when both the $h\nu_{peak}$ and t_{break} correlations with E_{iso} are accounted for, the remaining scatter in the latter quantity is remarkably small, less than a factor of 2 (e.g. Nava et al, 2006). This suggests that some quantity in GRBs is universal. The pure viewing angle interpretation of the Amati correlation posits that it is both the jet energy and spectral peak that vary little from one GRB to the next, while modest random variation in opening angle is acceptable and systematic variation of the opening angle with $E_{\gamma,iso}$ is not implied. The most natural underlying explanation of why this should be the case, we suggest, is that baryonic contamination is too small to affect the quantity of primary γ -ray emission (e.g. Levinson & Eichler, 1993; Eichler & Levinson 2000), and that, therefore, neither should the efficiency of internal shocks affect the overall energy output in γ -rays. By contrast, the non-thermal component of the prompt γ -ray emission, which does indeed vary

considerably among GRBs, may well depend on such factors. Similarly, the details of the erratic behavior of the light curve, in which there is considerably variety, may well depend on the less predictable aspects of GRB such as the internal shocks and baryon contamination.

This research was supported by the Israel-US Binational Science Foundation, an Israel Science Foundation Center of Excellence Award, and the Arnow Chair of Theoretical Physics.

REFERENCES

- Amati, L. et al., 2002, A&A, 390, 81
- Blandford, R.D. & Pyne, D.G. 1982, MNRAS, 199, 883
- Eichler, D. 1994, ApJS, 90, 877
- Eichler, D. & Granot, J. 2006, ApJ, in press (astro-ph/0509857)
- Eichler, D. & Jontof-Hutter, D. 2005, ApJ, 635, 1182
- Eichler, D. & Levinson A., 2000, ApJ, 529, 146
- Eichler, D. & Levinson A., 2004, ApJ, 614, L13
- Frail, D. A. et al., 2001, Ap. J. 562, L55
- Freedman, D.L., and Waxman, E., 2001 ApJ, 547, 922
- Ghirlanda, G., Ghisellini, G. & Lazzati, D. 2004, ApJ, 616, 331
- Lamb, D.Q., et al. 2004, NewA Rev., 48, 459
- Levinson, A. & Eichler, D. 1993, ApJ, 418, 386
- Levinson, A. & Eichler, D. 2005, ApJ, 629, L13
- Liang, E. & Zhang B. 2005, ApJ, 633, 611
- Lloyd-Ronning, N., and Zhang, B., 2004, ApJ, 613, 477
- Meszáros, P. Rees, M.J., & Wijers, R.A.M.J. 1998, 499, 301
- Nava, L. et al. 2006, A&A, in press
- Nousek, J.A., et al. 2005, submitted to ApJ (astro-ph/0508332)

Piro, L., et al. 2005, ApJ, 623, 314

Rees, M.J. & Meszaros, P. 2005, ApJ, 628, 847

Yamazaki, R. et al., 2004, ApJ 606, L33